

The Identification of Long Memory Process in the Asean-Four Stock Markets by Fractional and Multifractional Brownian Motion

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ABSTRACT

This research identifies the presence of long memory given return series of the stock markets in the ASEAN-4 countries, namely, Indonesia, Malaysia, the Philippines and Thailand. Daily stock prices from 1994 to 2004, which were neither adjusted for dividends nor inflation, were employed in the study. The Global Hurst Parameter is estimated to provide an indication of long memory. The sliding window method, quarterly, and annual estimations of the Hurst parameter were also undertaken to explore further on the behavior of the financial time series in question. The computed Hurst parameters across the windowed data, quarterly, and annual data has been shown to be time-varying. Stock prices behavior in this region is mildly persistent indicated by the following Global Hurst Parameters: Indonesia, 0.61, Malaysia, 0.58, the Philippines, 0.59, Thailand, 0.59.

KEYWORDS: Long Memory Processes, Multifractional Brownian Motion, Fractional Brownian Motion, Asian Stock Markets.

I. INTRODUCTION

This research is an undertaking of indication for long memory process of a financial time series of stock prices. When the innovations of the time series of the rates of return are independent, the time series can be modeled as a Brownian motion (Bm) also known as the Wiener process. A series with long memory may be better modeled as a fractional Brownian motion (fBm). However, since markets such as stock prices experience some “quiet” periods which correspond to a high Hölder exponent and some “erratic” periods which corresponds to a low Hölder exponent. Thus, financial time series may be better modeled as a generalized fBm with time-varying Hölder exponent known as the Multifractional Brownian Motion (mBm) (Ayache, Peltier, Vehel, 2000).

The authors are interested in estimating the Hurst parameter which indicates the presence or absence of Long Range Dependence (LRD) or Long Memory in a given process (Beran, 1994; Ayache, Peltier, and Vehel, 2000). Explaining reasons behind the presence or absence of LRD and the contextual discussion of structures, development levels and issues attendant to each stock market are beyond the purview of this research.

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The financial literature is replete with technical and practical discussions about the Hurst parameter, which is an indicator of Long Range Dependence (LRD). The interest on this parameter has largely been motivated by the aim to understand the behavior of financial time series, particularly its predictability. Where security prices, for instance, become predictable, knowledgeable investors could be able to make substantial above average profits on this basis, and, thus, knowing the behavioral component of the financial time series may have its benefits in terms of optimizing asset returns. Interestingly, the question of predictability is also part of the greater question of efficiency and accordingly the validity of the efficient market hypothesis (EMH), and the competitiveness of the stock market (Fama and French, 1988).

This research endeavors to identify or provide an indication of the presence of long memory in the stock market of four emerging ASEAN countries, namely, Indonesia, Malaysia, the Philippines, and Thailand collectively known as the ASEAN-4 countries. These countries were chosen on the basis of a very comparative level of economic structure and development with the Philippines.

Aside from financial time series data, there are physical phenomena that exhibit LRD in such as the annual minimum level of the Nile River and Data Communications Traffic (Beran, 1994). In addition, there is active research in the area of Image Analysis focuses on multidimensional extensions LRD in describing image textures (Pesquet-Popescu and Vehel, 2002).

II. THEORETICAL FRAMEWORK

A. Multifractional Brownian Motion: A Generalization of the Fractional Brownian Motion²

The mBm denoted as $\{B_{H(t)}(t), t \in \mathbb{R}\}$ is a stochastic fractal process (Pelier and Vehel, 1995) that generalizes the fBm (Mandelbrot and Van Ness, 1968) with a time-varying Hurst Parameter $H(t) \in (0,1)$, $t \in \mathbb{R}$. The mBm is represented as an integral form given as

$$B_{H(t)}(t) = \frac{\sigma}{\Gamma(H(t) + \frac{1}{2})} \left\{ \int_{-\infty}^0 \left[(t-s)^{H(t)-\frac{1}{2}} - (-s)^{H(t)-\frac{1}{2}} \right] dB(s) + \int_0^t \left[(t-s)^{H(t)-\frac{1}{2}} \right] dB(s) \right\} \quad (1)$$

where $B(s)$ is the standard Bm, that is, a Bm that has a zero mean and unit variance and $\sigma^2 = \text{var}(B_{H(t)}(t))|_{t=1}$ and the stochastic integral is taken in the mean-square sense (Peltier and Vehel, 1995). The fBm is a special case of the mBm with a fixed Hurst Parameter $H(t) = H$ and finally, the Bm is a special case of the fBm with a Hurst Parameter $H(t) = \frac{1}{2}$.

² This is taken largely from Ayache, Peltier, and Vehel (2000).

The mBm is a non-stationary Gaussian process and in general neither possesses independent nor stationary increments. In contrast, the fBm has stationary increments and in general not have independent increments except for the special case of the Bm.

The mBm has a Hölder exponent $\alpha(t)$ given as

$$\alpha(t) = \sup \left\{ \lim_{t \rightarrow 0} \frac{B_{H(t+\Delta t)}(t+\Delta t) - B_{H(t)}(t)}{|\Delta t|^\alpha} = 0 \right\} \stackrel{a.s.}{=} H(t). \quad (2)$$

In the case of the fBm, Hölder exponent

$$\alpha(t) \stackrel{a.s.}{=} H \quad (3)$$

The Hölder exponent can be interpreted as follows: “large” $\alpha(t)$ indicates a “smooth” trajectory of $B_{H(t)}(t)$ at time t while a “small” $\alpha(t)$ indicates a “rough” trajectory of $B_{H(t)}(t)$ at time t .

In addition, the following statistical characterizations are given as follows mBm are given as follows:

$$E[B_{H(t)}(t)] = 0 \quad (4)$$

$$\begin{aligned} \text{cov}[B_{H(t)}(t), B_{H(s)}(s)] &= \frac{\sigma^2}{2} \sqrt{\frac{\Gamma(2H(t)+1)\Gamma(2H(s)+1)\sin\left(\frac{\pi H(t)}{2}\right)\sin\left(\frac{\pi H(s)}{2}\right)}{\Gamma(H(t)+H(s)+1)\sin\left(\frac{\pi(H(t)+H(s))}{2}\right)}} \left(|t|^{H(t)+H(s)} + |s|^{H(t)+H(s)} - |t-s|^{H(t)+H(s)}\right) \\ &\quad (5) \end{aligned}$$

$$\text{var}[B_{H(t)}(t)] = \frac{\sigma^2}{2} |t|^{2H(t)} \quad (6)$$

Hence, from (4) to (6) following statistical characterizations of the fBm as a special case of the mBm are given as follows (Manolakis, Ingle, and Kogon, 2000):

$$E[B_H(t)] = 0 \quad (7)$$

$$\text{cov}[B_H(t), B_H(s)] = \frac{\sigma^2}{2} \left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right) \quad (8)$$

$$\text{var}[B_H(t)] = \frac{\sigma^2}{2} |t|^{2H} \quad (9)$$

The computed variances in (6) and (9) are well-known as the power law.

B. Long Range Dependence³

Let $\{X(k), k \in Z\}$ be at least a weakly stationary or wide sense stationary process (WSS), and then its auto-covariance is given as

$$\gamma_{x,x}(k) = \text{cov}[X(n), X(n-k)] \quad k, n \in Z \quad (10)$$

and its normalized auto-covariance function is given as

$$\rho_{x,x}(k) = \frac{\gamma_{x,x}(k)}{\gamma_{x,x}(0)} = \text{corr}[X(n), X(n-k)] \quad k, n \in Z \quad (11)$$

which is also referred to as the autocorrelation function (ACF). The stochastic process has long range dependence (LRD) also known as persistence if $\rho_{x,x}(k)$ is infinitely absolutely summable, that is,

$$\sum_{k=-\infty}^{\infty} |\rho_{x,x}(k)| = \infty \quad (12)$$

which is manifested it when $\rho_{x,x}(k)$ slowly decays to zero as $k \rightarrow \infty$. On the other hand, short (intermediate) range dependence (SRD) also known as anti-persistence if $\rho_{x,x}(k)$ is absolutely summable, that is,

$$\sum_{k=-\infty}^{\infty} |\rho_{x,x}(k)| < \infty \quad (13)$$

which is manifested it when $\rho_{x,x}(k)$ rapidly decays to zero as $k \rightarrow \infty$.

C. First Order Increments⁴

The Fractional Gaussian Noise (fGn) denoted as $\{G_H(k), k \in Z\}$ is defined as the first ordered sampled increment of the fBm given as

$$G_H(k) = B_H(k) - B_H(k-1) \quad k \in Z \quad (14)$$

The fGn has the following statistics given as follows:

$$E[G_H(k)] = 0 \quad (15)$$

$$\gamma_{G_H,G_H}(k) = \text{cov}[G_H(n), G_H(n-k)] = \frac{\sigma^2}{2} (|k+1|^{2H} + |k-1|^{2H} - 2|k|^{2H}) \quad (16)$$

³ This is taken largely from Brockwell and Davis (1987).

⁴ This is taken largely from Manolakis, Ingle, and Kogon, (2000).

$$\rho_{G_H, G_H}(k) = \text{corr}[G_H(n), G_H(n-k)] = \frac{1}{2}(|k+1|^{2H} + |k-1|^{2H} - 2|k|^{2H}). \quad (17)$$

The fGn is a WSS process manifested from its mean and the covariance statistics. Thus, the characterization of the non-stationary fBm can be conveniently obtained from the fGn. By examining the asymptotic behavior of (17), for $H = \frac{1}{2}$, $\rho_{G_H, G_H}(k) = \delta_k$ where δ_k is the Kronecker Delta Function. On the other hand, for $H \neq \frac{1}{2}$, $\rho_{G_H, G_H}(k) = O(k^{2H-2})$. Hence, from (12) and (13), the fBm has a SRD if $0 < H \leq \frac{1}{2}$ and it has a LRD if $\frac{1}{2} < H < 1$.

The mBm analogy of the fGn which we call as Multifractional Gaussian Noise (mGn) denoted as $\{G_{H(k), H(k-1)}(k), k \in Z\}$ is defined as the first ordered sampled increment of the mBm given as

$$G_{H(k), H(k-1)}(k) = B_{H(k)}(k) - B_{H(k-1)}(k-1) \quad k \in Z. \quad (18)$$

Since the mGn in general does not have stationary increments, then the mGn is not a stationary process. Some characterization of this process is given in the paper of Ayahe, Cohen, and Vehel, (2000).

III. METHODOLOGICAL FRAMEWORK

This research identifies the Hurst parameter by modeling the time series as the fBm and mBm.

A. Sliding and Growing Windows

The sliding window consists of a fixed number of samples denoted by N such that the number of samples between adjacent windows is fixed denoted by M . Let $W_i(k)$ be the i^{th} sliding window derived from the return series $X(k)$, then,

$$W_i(k) = X((i-1)M + k), \quad 1 \leq k \leq n \quad (19)$$

and the window derived from the last $r < N$ samples of $X(k)$ is discarded. If $M < N$, then the windows are intersecting on the other hand, if $M \geq N$ then windows are non-intersecting. There are trade-offs in the selection of the pairs (M, N)

$\frac{M}{N}$ - "small": **Tightly Intersecting Windows.**

Larger number of windows hence increases computational time in computing $H(t)$ throughout the time series. The resulting trajectory of $H(t)$ is smooth.

$\frac{M}{N}$ - “large”: ***Loosely Intersecting Windows.***

Smaller number of windows hence computational time in computing $H(t)$ throughout the time series. The resulting trajectory of $H(t)$ is discretized.

N - “small”: ***Small Data Window.***

There may be not enough points to reliably estimate the Hurst Parameter in the range $H(t) \in (0,1)$.

N - “large”: ***Large Data Window.***

Hurst Parameter estimation may be flattened out; the Hurst Parameter may vary throughout time.

Given the potential tradeoffs, the choice of pairs M and N in this study is given as $M = 16$ and $N = 128$.

On the other hand, the increasing or growing window is a data window that consists of data points gathered from some reference or base time t_0 interval up to some present time interval t . As time progresses, the number of points in the data window increases for which the nomenclature is drawn.

A. fBm Modeling

The Hurst parameter is estimated using a growing window in a Sliding Window Increments (from the Sliding Window), Quarterly, and Annualized basis to examine the convergence of Global Hurst Parameter. The downside to this approach is the computational complexity involved in the estimation process.

B. mBm Modeling

The Hurst Parameters of the following windowed time series are estimated using the Sliding Window Data, Quarterly Data, and Annual Data. To simplify the analysis, it is assumed that the Hurst Parameter throughout a fixed window is constant or at most slowly varying, hence we can model the windowed data as an fBm. Since the Hurst Parameters across different windows vary, we can regard this model as a discretized mBm model across the time series.

C. Pre-processing

For this paper, the rate of return is the first difference of the natural log of the index series to remove the market trend from the return calculation:

$$Y(k) = \log \left[\frac{X(k)}{X(k-1)} \right] \quad (20)$$

where, $X(k-1)$ is the index at time k and $X(k-1)$ is the index at time $k-1$.

D. Hurst Parameter Estimation⁵

Let $\{X(t), t \in \mathbb{R}\}$ be an fBm with Hurst Parameter, H then by self-similarity of $X(t)$ we obtain

$$E\left[\left|X(k + \Delta k) - X(k)\right|^2\right] = c_1 (\Delta k)^{2H} \quad (22)$$

where $c_1 = E\left[\left|\Delta X(1)\right|^2\right]$ and $\Delta k, k \in \mathbb{Z}$. Taking the logarithm gives us

$$\log E\left[\left|X(k + \Delta k) - X(k)\right|^2\right] = \log c_1 + 2 \log \Delta k \cdot H \quad (23)$$

From (23), H can be estimated using linear regression. However, this estimator can be prone to outliers. As an alternative, by the self-similarity of $X(t)$

$$E\left[\left|X(k + \Delta k) - X(k)\right|\right] = c_2 (\Delta k)^H \quad (24)$$

where $c_2 = E\left[\left|\Delta X(1)\right|\right]$ and $\Delta k, k \in \mathbb{Z}$. Taking the logarithm gives us

$$\log E\left[\left|X(k + \Delta k) - X(k)\right|\right] = \log c_2 + \log \Delta k \cdot H \quad (25)$$

From (25), H can be estimated using linear regression.

E. Triangulation

A triangulation will be done to understand further and validate some claims regarding and surrounding the Hurst parameter. First, the ACF for the sliding windows, quarterly, and annual windows will be presented to show and provide evidence that the return series exhibit long memory. Then, the sliding window, quarterly, and annual Hurst parameter estimates validate the claim that the Hurst parameter is time varying. Thirdly, the Global Hurst parameter is estimated using an increasing window to see the process of convergence to the value of the Global Hurst parameter and, again, to provide a more compelling evidence that the series in question do exhibit long memory.

IV. DATA

This research makes use of daily observations of nominal stock market closing for the period 1994 to 2004 for the four ASEAN stock markets, namely, the Philippine Stock Exchange (PSE), the Jakarta Stock Exchange (JSE), the Kuala Lumpur Stock Exchange (KLSE), and the Stock Exchange of Thailand (SET). The PSE Composite Index, the JSE Composite Index, the KLSE Composite Index and the SET Composite Index are all expressed in local currencies and neither adjusted for dividends nor inflation. Throughout the rest of this paper, the source of the computed figures were computed by the authors. In addition, computed quarterly and annual figures are given in the Appendix. Figure 1 below reveals some trend about the return series of the four stock exchanges. The return series of

⁵ This is taken largely from Manolakis, Ingle, and Kogon, (2000).

the four exchanges exhibit volatile returns in the short term and an evident trend in the long run. All, except for Indonesia, exhibit decreasing trends in varying extent considering the period. On the other hand, Table 1 features descriptive statistics for the return series $X(k)$.

Figure 1: Stock Index Returns, 1994 to 2004.

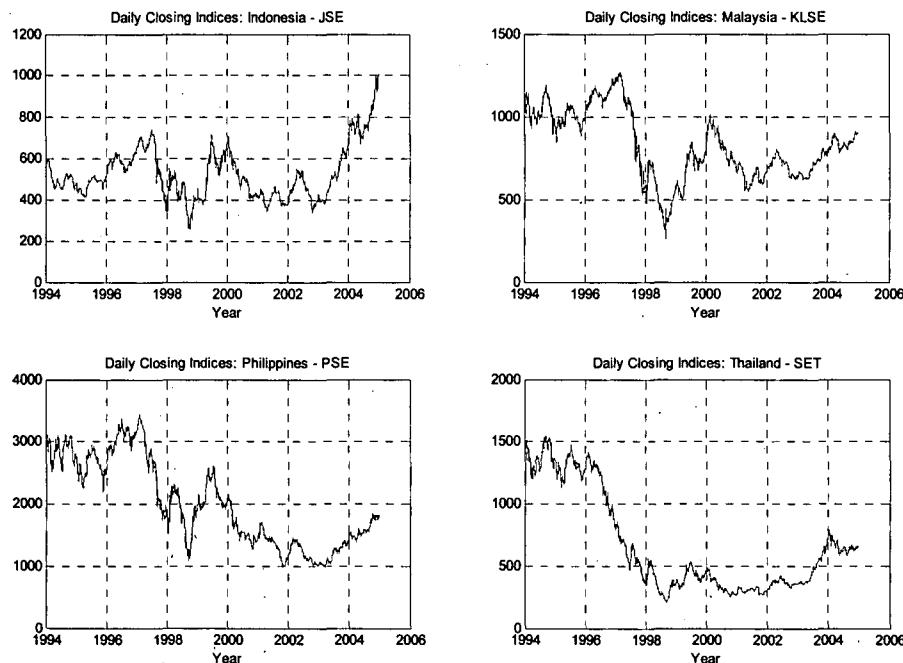


Table 1. Four Moments for the ASEAN-4

Country	Mean	Variance	Normalized Skewness	Normalized Kurtosis -3
Indonesia	526.93	15759.25	0.9249	0.8954
Malaysia	828.26	43552.07	0.0766	-0.6930
Philippines	2006.62	489394.54	0.3168	-1.2525
Thailand	670.97	162953.84	0.8620	-0.8056

All series means are positive, and all return series exhibited non-zero excess normalized skewness. The normalized kurtosis coefficients are all different from 3 (the value for a Gaussian Distribution). Any formal Gaussianity test result is not reported since, the series is clearly non-Gaussian. One can also immediately posit that all return series are non-Gaussian based from the normalized kurtoses and the non-zero normalized skewnesses. The robustness of the Hurst Parameter estimation in the midst of non-Gaussianity of the time series will be assessed in the later part of the paper.

V. EMPIRICAL RESULTS AND DISCUSSION

A. Long Range Dependence Examination via the Autocorrelation Function Plots

The ACF across discrete time can be visually represented in a 3D plot or perhaps using Hinton graph which borrowed from Artificial Neural Networks (Bishop, 1995) where the neuron is replaced by ACF lags and the network layers can be replaced by time. On the other hand, although the ACF is discrete-space process by presenting it in a contour plot has several advantages over the previous two methods discussed. First, it is more convenient means of visualizing the behavior of the ACF as both lags and time progresses and second, it uses less space.

The contour of the ACF plots presented below (Figures 2a to 2c) and difference of ACF level in each succeeding contour line is 0.10. The contour plots show a rapidly ACF as lags progresses. In general, the most ACF values are positive with a few patches of the contour which located at later lags that have negative ACF. We suspect that a mildly persistent financial time series behavior

Figure 2a: ACF (Sliding Window): 1994 to 2004

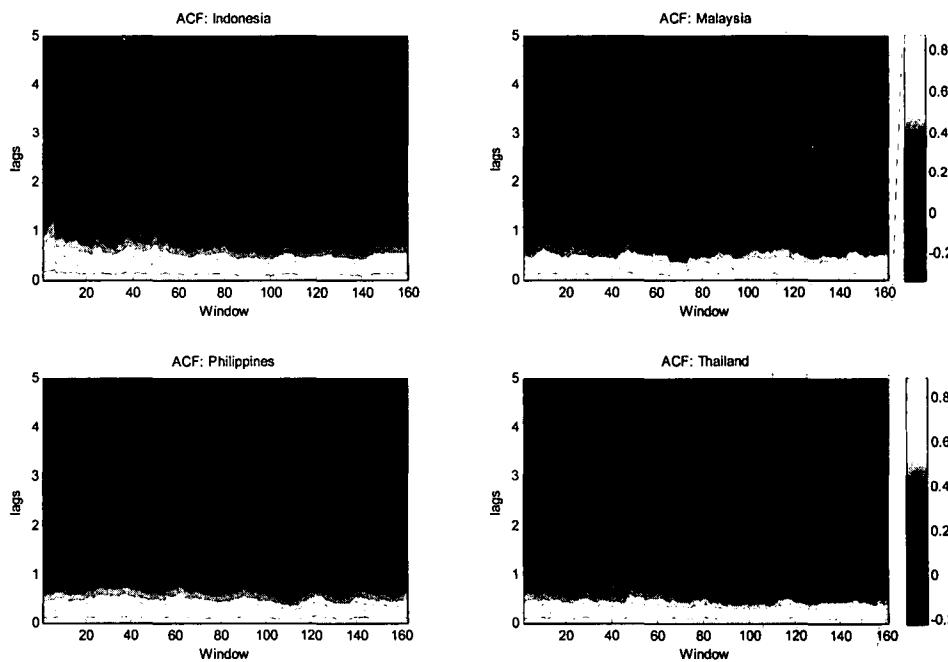


Figure 2b: ACF (Sliding Window): Quarterly

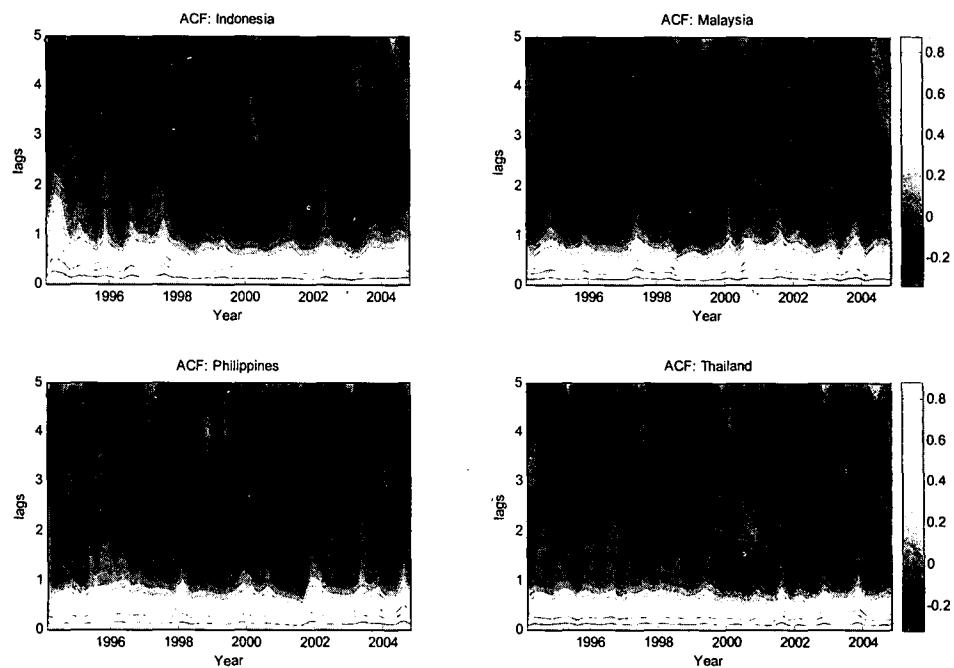
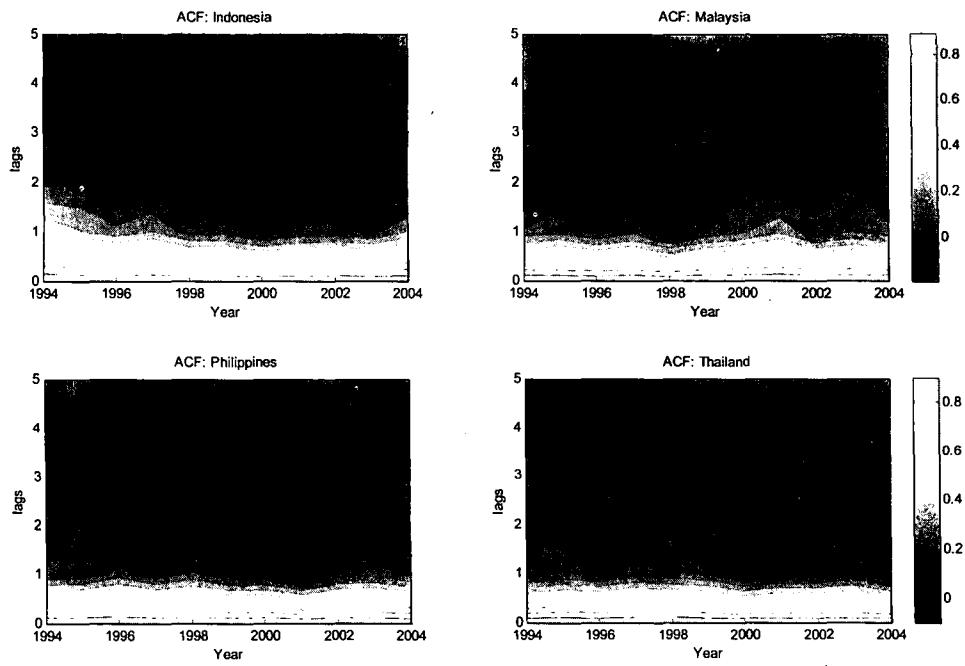


Figure 2c: ACF (Sliding Window): Annual



B. Time Varying Hurst Parameter Estimation

The claim that the long memory is time varying is investigated by estimating the Hurst parameter using the sliding window approach and having quarterly and yearly estimates. This allows the researchers to see the rather volatile behavior of the parameter as we move from quarterly to annual estimations. Somehow, the rather volatile character of the financial time series given the sliding-window and quarterly outcomes eases when one deals with the yearly characterization (Figures 3a to 3c).

Figure 3a: Hurst Parameter: Sliding Window

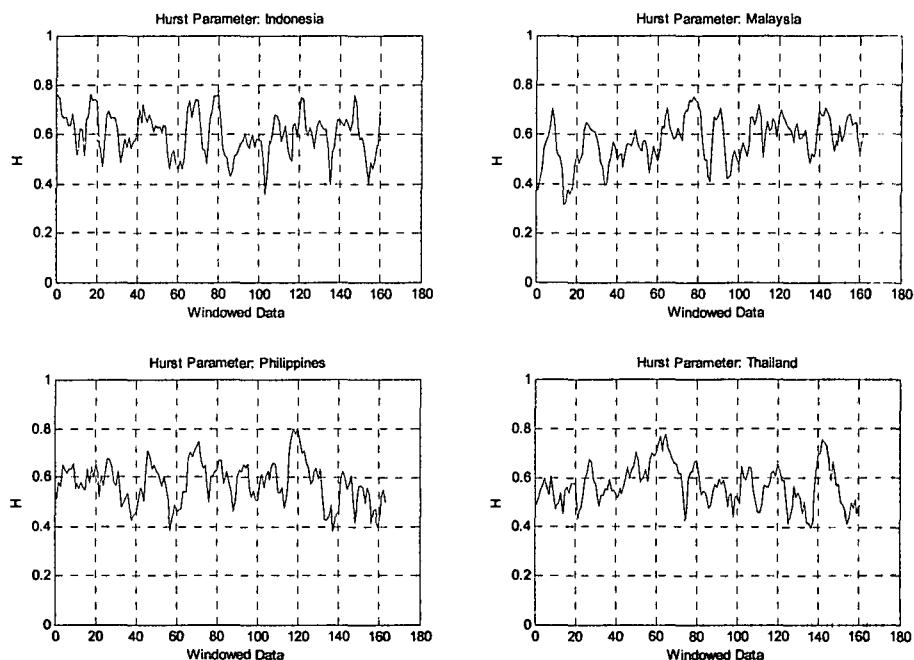
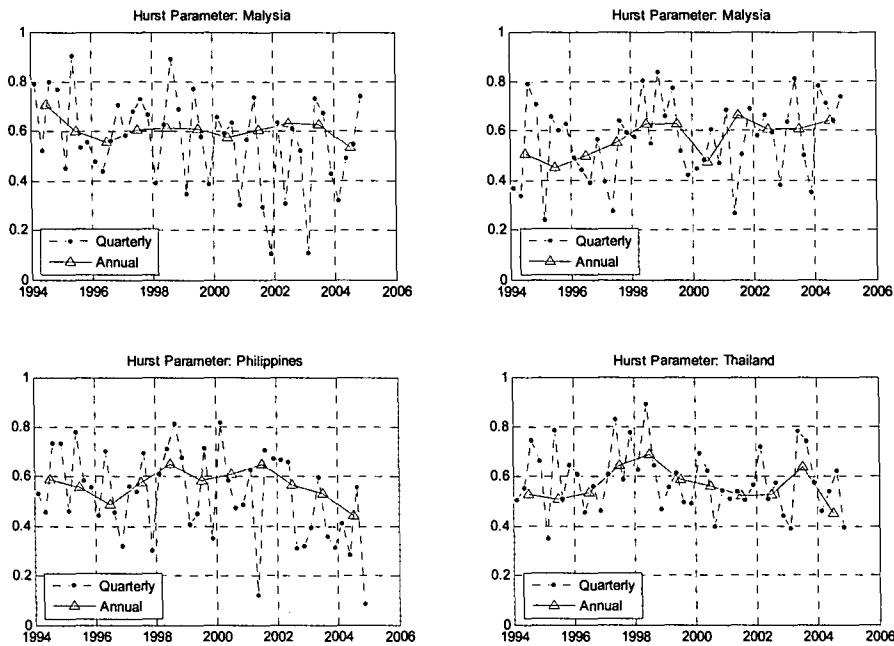


Figure 3b: Hurst Parameter: Quarterly and Annual Basis



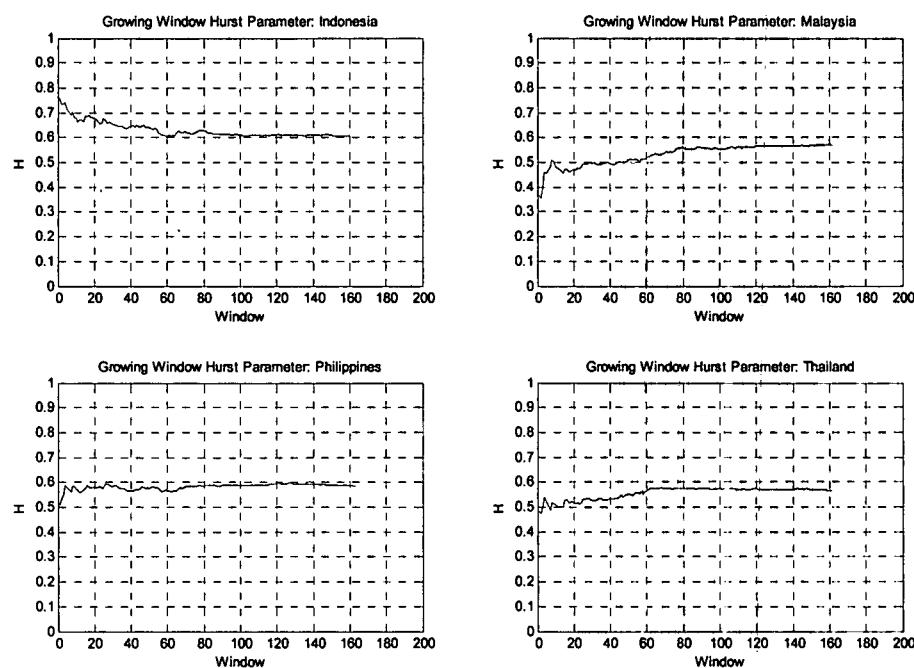
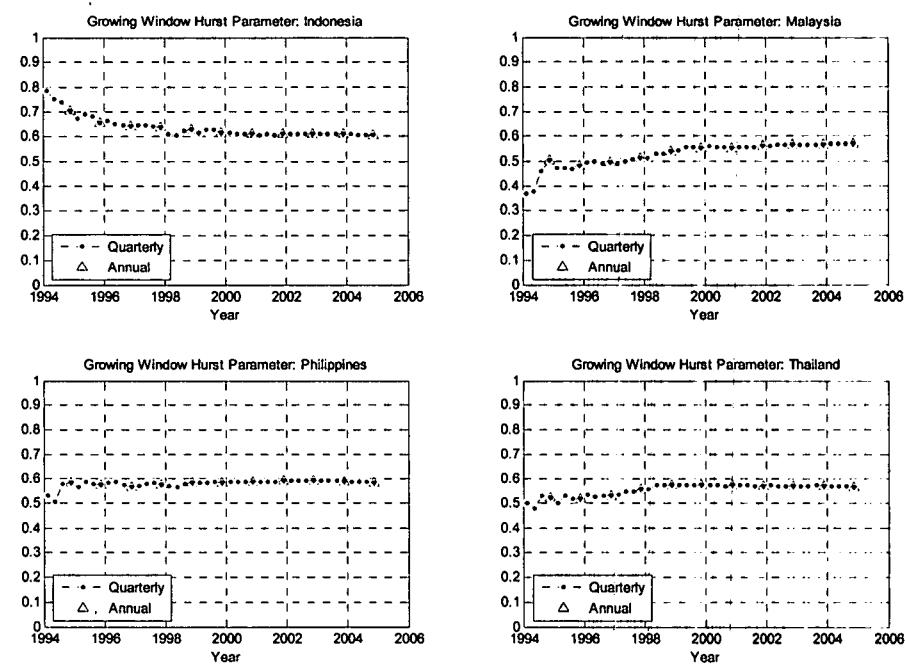
C. Global Hurst Parameter Estimation and Convergence

The identified Global Hurst parameter characterizes the entire financial in terms of long memory process. Figures 4a to 4c shows that each financial time series converges to the values of the Global Hurst parameter from a growing window with a base year 1994. Table 2 reveals the estimated Global Hurst parameter that characterizes each financial time series. Indonesia has the highest at 0.61, while Malaysia, the Philippines, Thailand have 0.58, 0.59, and 0.59 respectively.

The estimated Global Hurst parameters ASEAN-4 countries are all above 0.50 and around 0.60 suggests that the financial time series of in these countries have rather weakly to mildly persistent behavior.

Table 2. Identified Global Hurst Parameter: 1994-2004

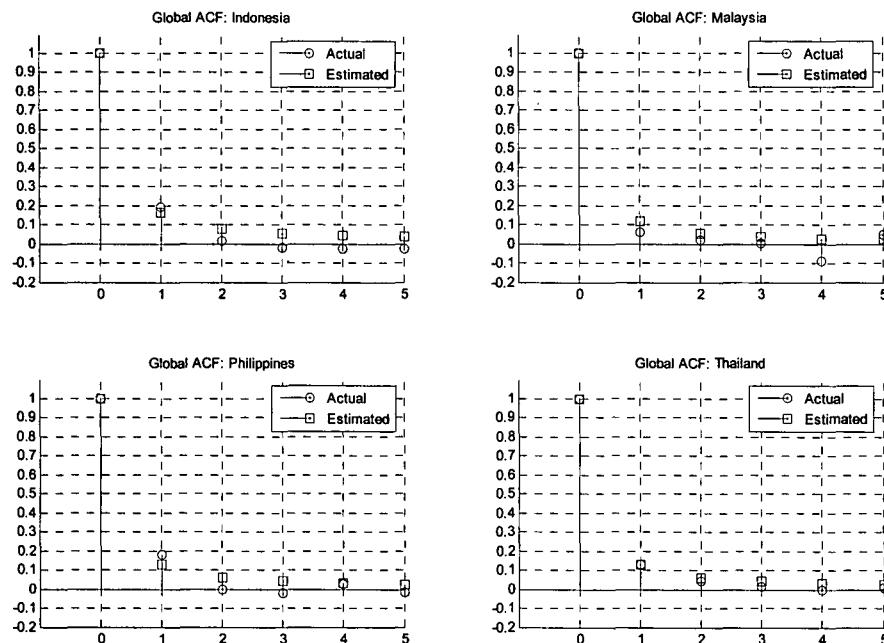
Country	Global Hurst Parameter
Indonesia	0.61
Malaysia	0.58
Philippines	0.59
Thailand	0.59

Figure 4a: Growing Window: Sliding Window**Figure 4b: Growing Window: Quarterly and Annual Data**

D. Global ACF and Performance Assessment

From the estimated Hurst parameters, one can estimate the ACF using (17) for a given data window. As the data window increases, discrepancy between the actual ACF and the estimated ACF subsides. Figure 5 shows the plot of the actual Global ACF from the estimated ACF which are relatively close to each other. This suggests robustness of the estimated Hurst parameter despite the non-Gaussianity of the time series.

Figure 5: Global ACF Comparison



VI. CONCLUSION

The estimated Global Hurst parameters of the four ASEAN-4 countries provide an indication of weakly to mildly persistent stock prices, considering the return series. Since the four estimated Global Hurst Parameters are close to each other, hence the degree of persistence of stock prices in these countries are comparable to each other by viewing the entire time series. In addition, in the presence time variations in the Hurst Parameters, there are variations in the degree of persistence (or anti-persistence) by considering of the local portion of the time series when we examine the ACF and the Hurst Parameters in Figures 2 and 3 respectively.

It should come to bear that this is an identification exercise and not an exploration of reasons behind the outcome, and, in addition, that the Hurst Parameter only measures the degree of dependence, but still fails to reflect how market prices adjust to shocks. Thus, caution is needed in interpreting the values of the reported Global Hurst Parameter. Moreover, the fact that persistence can be time varying casts a cloud of caution with regard to the findings. One cannot posit whether the above results are temporary phenomena or

whether they apply universally. Unfortunately, available data do not allow for such an investigation, as they do not reach far back enough.

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